IMPACT: International Journal of Research in Engineering & Technology (IMPACT: IJRET)
ISSN(E): 2321-8843; ISSN(P): 2347-4599
Vol. 3, Issue 9, Sep 2015, 31-42

ANT COLONY OPTIMISATION APPROACH TO SELECTIVE HARMONIC ELIMINATION IN MULTILEVEL INVERTER

ADEYEMO, I. A.[1], FAKOLUJO, O. A.[2] & ADEPOJU, G. A.[3]

[1, 3] Department of Electronic & Electrical Engineering, Ladoke Akintola University of Technology, PMB, Ogbomoso, Oyo State, Nigeria

[2] Department of Electronic & Electrical Engineering, University of Ibadan, Ibadan, Oyo State, Nigeria

ABSTRACT

In Selective Harmonic Elimination-Pulse Width Modulation (SHE-PWM) method, low order harmonics are eliminated, while the fundamental harmonic is obtained at the desired value. In this paper, Variable Sampling Ant Colony Optimization (SamACO) algorithm with random initial values is proposed for solving the transcendental nonlinear equations known as Selective Harmonic Elimination (SHE) equations that characterize the selected harmonics in an 11-level inverter. The algorithm is a continuous (combinatorial) optimization algorithm that is based on the food foraging behavior of ants in a swarm. The dynamic exploitation and random exploration operators in the algorithm ensure both accuracy and convergence to global optima. Fast Fourier Transform (FFT) analysis of the synthesized voltage waveform reveals the complete elimination of the $5^{th}, 7^{th}, 11^{th}$ *and* 13^{th} harmonics as their values tend towards zero. Both computational and MATLAB simulation results show that the proposed method is highly efficient for elimination of the selected low order harmonics as well as minimization of the total harmonic distortion (THD).

KEYWORDS: Multilevel Inverter, Samaco, Selective Harmonic Elimination (SHE), and THD

INTRODUCTION

A multilevel voltage source inverter is a power electronic system that synthesizes a nearly sinusoidal output voltage from several DC voltages using pulse width modulation (PWM) technique. Due to the smaller voltage steps in the output staircase waveform, multilevel inverter has many advantages over the traditional among which are: improved power quality, low switching losses, lower dv/dt stresses on the load, lower electro-magnetic interference (EMI) and ability to attain a higher voltage without the use of transformer [1]. The use of multilevel inverter is prevalent in industrial applications such as drives, Flexible AC Transmission Systems (FACTS), Hybrid Electric Vehicle (HEV), High Voltage Direct Current (HVDC) lines.

The concept of multilevel inverters was actually developed from the idea of step approximation of sinusoid [2]. Basically, there are three main multilevel topologies. These are Diode-Clamped Multilevel Inverter [3], Capacitor-Clamped Multilevel Inverter [4], and Cascaded H-bridge Multilevel Inverter with separate DC sources [5]. Varieties of each topology as well as hybrid of the fundamental topologies such as Generalized P2 Converter, Mixed-Level Hybrid Converter, Asymmetric Hybrid converter have been developed but with the same underlying principle[6,7].

Several pulse width modulation techniques used in conventional two-level inverter have been modified and deployed in multilevel inverters. These include Sinusoidal Pulse Width Modulation (SPWM), Selective Harmonic

Elimination (SHE) method, Space Vector Control (SVC), and Space Vector Pulse Width Modulation (SVPWM) [6, 7]. Selective Harmonics Elimination (SHE) method at fundamental switching frequency however, arguably gives the best result because of its high spectral performance and considerably reduced switching loss. The main challenge associated with the SHE method is how to obtain the analytical solutions of the transcendental nonlinear equations that characterize the selected harmonics in multilevel inverter.

The convergences of the classical derivative-dependent solvers such as Newton Raphson method [8] are highly sensitive to arbitrarily chosen initial values of the solution. Another approach uses Walsh functions [9] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angle. The method results in a set of algebraic matrix equations and the calculation of the optimal switching angles is a complex and time-consuming operation. Chiasson *et al* [10] proposed a method based on Elimination theory using resultants of polynomials to determine the solutions of the SHE equations. A difficulty with this approach is that as the number of levels increases, the order of the polynomials becomes very high, thereby making the computations of solutions of these polynomial equations very complex.

Population-based Evolutionary Algorithms (EAs) such as Genetic Algorithm (GA) [11]-[13], Ant Colony System (ACS) [14], Bee Algorithm (BA) [15], Particle Swarm Optimization (PSO) [16] have been used to solve SHE equations. EAs are simple, derivative free and can be used for problems with any number of levels. They are also successful in locating the optimal solutions, but they are usually slow in convergence and require much computing time.

MULTILEVEL INVERTERS

- **Cascaded H-bridge Inverter**

Among multilevel inverter topologies, cascaded H-bridge inverter requires the least number of components. Its modular structure as well as circuit layout flexibility makes it suitable for high voltage and high power applications. Cascaded H-bridge multilevel inverter is formed by connecting several single-phase H-bridge inverters in series as shown in Figure 1. The number of output voltage levels in a cascaded H-Bridge inverter is given by $N = 2S + 1$, where S is the number of H-bridges per phase connected in cascade. By different combinations of the four switches S_1, S_2, S_3, and S_4 shown in the Figure 1, each H-bridge switch can generate a square wave voltage waveform on the AC side. To obtain $+V_{dc}$, switches S_1 and S_4 are turned on, whereas $-V_{dc}$ can be obtained by turning on switches S_2 and S_3. By turning on S_1 and S_2, or S_3 and S_4, the output voltage is zero.

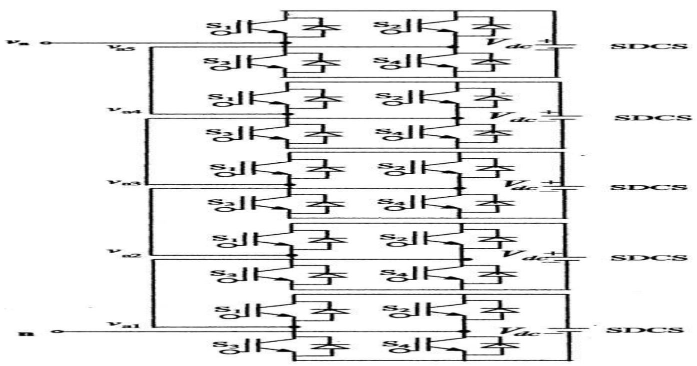


Figure 1: Configuration of an 11-Level Single-Phase Cascaded H-Bridge Multilevel Converter

The outputs of H-bridge switches are connected in series such that the synthesized AC voltage waveform shown in Figure 2 is the summation of all voltages from the cascaded H-bridge cells [8, 10].

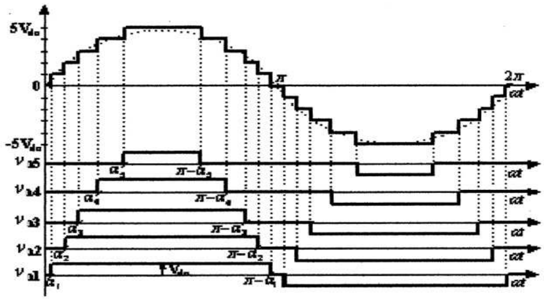


Figure 2: Output Voltage Waveform of an 11-Level Inverter

- **Selective Harmonic Elimination PWM**

Generally, any periodic waveform such as the staircase waveform shown in Fig. 2 can be shown to be the superposition of a fundamental signal and a set of harmonic components. By applying Fourier transformation, these components can be extracted since the frequency of each harmonic component is an integral multiple of its fundamental [17].

Assuming a quarter wave symmetry and the equal amplitude of all DC sources, the Fourier series expansion of the staircase output voltage waveform shown in Fig. 2 is given by equation (1).

$$V(\omega t) = V_n(\alpha)\sin(n\omega t) \tag{1}$$

Where

$$V_n(\alpha) = \frac{4V_{dc}}{n\pi}\sum_{k=1}^{S}\cos(n\alpha_k)\text{, for odd n} \tag{2}$$

$$V_n(\alpha) = 0\text{, for even n} \tag{3}$$

In three-phase power system, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages as a result only non-triplen odd harmonics are present in the line-to-line voltages 8]

Combining equations (1), (2) and (3),

$$v(\omega t) = \sum_{n=1,3,5...}^{\infty}\frac{4V_{dc}}{n\pi}(\cos(n\alpha_1)+\cos(n\alpha_2)+...+\cos(n\alpha_S))\sin n\omega t) \tag{4}$$

Subject to $0 < \alpha_1 < \alpha_2 < ...\alpha_S \leq \pi/2$

Where, *S* is the number of switching angles and *n* is the harmonic order. Generally, for *S* number of switching angles, one switching angle is used for the desired fundamental output voltage V_1 and the remaining (*S*-1) switching angles are used to eliminate certain low order harmonics that dominate the Total Harmonic Distortion (THD) such that equation (4) becomes

$$V(\omega t) = V_1\sin(\omega t) \tag{5}$$

From equation (4), the expression for the fundamental output voltage V_1 in terms of the switching angles is given by

$$V_1 = \frac{4V_{dc}}{\pi}\left(\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_S)\right) \quad (6)$$

The relation between the fundamental voltage and the maximum obtainable fundamental voltage V_{1max} is given by modulation index. The modulation index, m_i, is defined as the ratio of the fundamental output voltage V_1 to the maximum obtainable fundamental voltage V_{1max}. The maximum fundamental voltage is obtained when all the switching angles are zero [8]. From equation (6),

$$V_{1\max} = \frac{4SV_{dc}}{\pi} \quad (7)$$

$$\therefore m_i = \frac{V_1}{V_{1\max}} = \frac{\pi V_1}{4SV_{dc}}$$

Hence,

$$V_1 = m_i\left(\frac{4SV_{dc}}{\pi}\right) \quad \text{for } 0 < m_i \le 1 \quad (8)$$

To develop an 11-level cascaded multilevel inverter, five SDCSs are required. The modulation index and switching angles that result in the synthesis of AC waveform with the least Total Harmonic Distortion (THD) can be found by solving the following transcendental nonlinear equations known as SHE equations that characterize the selected harmonics[8], [10]:

$$\frac{4V_{dc}}{\pi}(\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_5)) = V_1$$

$$\cos(5\alpha_1) + \cos(5\alpha_2) + ... + \cos(5\alpha_5) = V_5$$

$$\cos(7\alpha_1) + \cos(7\alpha_2) + ... + \cos(7\alpha_5) = V_7$$

$$\cos(11\alpha_1) + \cos(11\alpha_2) + ... + \cos(11\alpha_5) = V_{11}$$

$$\cos(13\alpha_1) + \cos(13\alpha_2) + ... + \cos(13\alpha_5) = V_{13} \quad (9)$$

In equation 9, V_5, V_7, V_{11}, and V_{13} are set to zero to in order to eliminate 5^{th}, 7^{th}, 11^{th} and 13^{th} harmonics respectively. The correct solution must satisfy the condition

$$0 \le \alpha_1 < \alpha_2 < ... < \alpha_5 \le \pi/2 \quad (10)$$

Equation (8) in equation (9) yields:

$$\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_5) = 5m_i \;\; \cos(5\alpha_1) + \cos(5\alpha_2) + ... + \cos(5\alpha_5) = 0 \;\; \cos(7\alpha_1) + \cos(7\alpha_2) + ... + \cos(7\alpha_5) = 0$$
$$\cos(13\alpha_1) + \cos(13\alpha_2) + ... + \cos(13\alpha_5) = 0 \quad (11)$$

Generally equation (11) can be written as

$$F(\alpha) = B(m_i) \tag{12}$$

The Total Harmonic Distortion (THD) is computed as shown in equation (13):

$$THD = \sqrt{\sum_{i=5,7,11,13,...}^{49} \left(\frac{V_i}{V_1}\right)^2} \tag{13}$$

ANT COLONY OPTIMIZATION (ACO)

The ant colony optimization is swarm intelligence based meta-heuristic algorithm that was inspired by the food foraging behavior of natural ants. It is a probabilistic technique for solving combinatorial optimization problems that can be reduced to finding good path through graphs. In their search for food, natural ants leave a trail of chemical substance called pheromone on the path they traverse in order to guide future ants toward optimal paths to food. The pheromone on the non-optimal paths evaporates with time while the pheromone on the near-optimal/optimal paths is reinforced, thus influencing more ants to follow the path [18].

In ACO, a population of agents (artificial ants) incrementally constructs solutions to combinatorial optimization problem by traversing a graph that encodes the optimization problem. Ant Colony system (ACS) algorithm, introduced by Dorigo and Gambardella [19] had been successfully deployed for discrete combinatorial optimization problems such as routing, and clustering. ACO algorithm has been extended to solving continuous combinatorial optimization problems using a variety of ACO algorithm called Variable Sampling Ant Colony Optimization (SamACO) algorithm [20]. SamACO algorithm offers an efficient incremental solution construction method based on the sampled values.

The basic idea behind SamACO algorithm is that a population of agents (artificial ants) incrementally constructs solution to the sampled combinatorial optimization problem. The construction phase is guided by heuristic information and existing pheromone, which holds information about parts of a solution that have led to good results in the previous generation of ants. By means of pheromone update and transition rules, ACO probabilistically select the components values to concentrate the search in the regions of high quality solutions.

The steps that are involved in the implementation of SamACO algorithm are as follows:

Initialization Step

The search space is bounded such that the decision variables (solution components) X_i has values $x_i \in [l_i, u_i]$, $i = 1,2,\ldots,S$, where l_i and u_i are the lower and upper bounds of the decision variables X_i respectively, and S is the number of decision variables. The initial values of the decision variables are randomly sampled in the feasible domain as follows:

$$x_i^{(j)} = l_i + \frac{u_i - l_i}{m + \vartheta}\left(j - 1 + rand_i^{(j)}\right) \tag{14}$$

Where m is the number of ants, ϑ is the exploitation frequency which controls the number of values to be sampled in the neighborhood of the best-so-far solution per iteration, $(m+\vartheta)$ is the initial number of candidate values for each decision variable x_i^j , rand is a random number uniformly distributed within $[0, 1]$, $i = 1, 2, \ldots, S$ and $j = 1, 2, \ldots, (m+\vartheta)$.

For each decision variable X_i , there are k_i sampled values $x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(k_i)}$ from the continuous domain $[l_i, u_i]$. Each solution component has associated a pheromone value, τ_i^j, $i = 1,2,\ldots,S$, $j = 1,2,\ldots,k_i$, and a component-pheromone matrix M can be generated. The pheromone value τ_i^j reflects the desirability of adding the component value x_i^j to the solution

$$M = \begin{bmatrix} \{x_1^{(1)}, \tau_1^{(1)}\} & \{x_2^{(1)}, \tau_2^{(1)}\} \ldots \{x_S^{(1)}, \tau_S^{(1)}\} \\ \{x_1^{(2)}, \tau_1^{(2)}\} & \{x_2^{(2)}, \tau_2^{(2)}\} \ldots \{x_S^{(2)}, \tau_S^{(2)}\} \\ \vdots & \vdots \quad \ddots \quad \vdots \\ \{x_1^{(k_1)}, \tau_1^{(k_1)}\} & \{x_2^{(k_2)}, \tau_2^{(k_2)}\} \ldots \{x_S^{(k_S)}, \tau_S^{(k_S)}\} \end{bmatrix} \tag{15}$$

Transition

The transition of the ants from one position to another is partially probabilistic and partially deterministic. An artificial ant k has a memory of the positions that it has already visited and the pheromone content at each location stored in a Tabu list T^k . The memory size of the Tabu list depends on the ant population size as well as the number of movement made by the ants. In general, if there are m ants making N movement, the size of the Tabu list is $(m \times N)$. The iteration index l_i^k of the variable value selected by ant k for the i th variable is:

$$l_i^k = \begin{cases} 1 & if \quad q \le q_o \quad and \quad j = j^* \\ 0 & if \quad q \le q_o \quad and \quad j \ne j^* \\ L_i^{(k)} & if \quad q > q_o \end{cases} \tag{16}$$

Where

$$j^* = \arg\max\{\tau_i^{(1)}, \tau_i^{(2)}, \cdots, \tau_i^{(m)}\} \tag{17}$$

$i = 1,2,\ldots,s$, $k = 1, 2,\ldots,m$, $q \in [0,1]$ is a uniform random value, and $q_o \in [0,1]$ is a threshold parameter that represents the relative preference for either exploitation or exploration.

Dynamic Exploitation

When $q \le q_o$, the ant chooses exploitation in the neighborhood of the solution set with the highest pheromone value from the m solutions generated in the previous iteration. The dynamic exploitation is used as a local search method to fine-tune the best-so-far solution. A radius r_i confines the search in the neighborhood of the best-so-far solution $x^{(0)} = (x_1^{(0)}, x_2^0, \cdots, x_s^0)$ to the interval $[x_i^{(0)} - r_i, x_i^{(0)} + r_i]$, $i = 1, 2, \cdots, S$. The values of the variables in the best-so-far solution set are randomly selected to be increased, unchanged or reduced as

$$\hat{x}_i = \begin{cases} \min\left(x_i^{(0)} + r_i.\sigma_i, u_i\right), & 0 \le q < \frac{1}{3} \\ x_i^{(0)}, & \frac{1}{3} \le q < \frac{2}{3} \\ \max\left(x_i^{(0)} - r_i.\sigma_i, l_i\right), & \frac{2}{3} \le q < 1 \end{cases} \tag{18}$$

Where $\sigma \in [0, 1]$.

The best-so-far solution set is then updated using elitism and generational replacement. The new solution set $\hat{x} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_S)$ is evaluated and used to replace the best-so-far solution set if there is an improvement in the result. The dynamic exploitation process is repeated for ϑ times, and the newly generated solution components are recorded as $x_i^{(j)}$, where $j = m+1, m+2, \cdots, m+g_i$, $i = 1,2,\ldots,S$. The number of solution components generated during the dynamic exploitation process is denoted by g_i. The radii are adaptively extended or reduced based on the exploitation result. If the best solution set produced by the exploitation process is better than the prevailing best-so-far solution set, the radii will be extended. Otherwise, the radii will be reduced.

$$r_i \leftarrow \begin{cases} r_i . v_e, & v_e > 1 \\ r_i . v_r, & 0 < v_r \leq 1 \end{cases} \tag{19}$$

Where v_e and v_r are the radius extension rate and the radius reduction rate, respectively. The initial radius value is given by:

$$r_i = \frac{u_i - l_i}{2m} \tag{20}$$

Random Exploration

When $q > q_o$, the ant resort to probabilistic exploration to select a random index $L_i^{(k)} \in \{0,1,\cdots,m+g_i\}$. Moving from position i, the ant chooses its next position j among the positions that have not been visited yet according to the probability distribution given as follows:

$$p_i^{(j)} = \frac{\tau_i^{(j)}}{\sum_{u=0}^{m+g_i} \tau_i^{(u)}}, \quad j = 0,\ 1, \cdots, m+g_i \tag{21}$$

The solution components of the worst x solution sets that are constructed by the ants in the previous iteration are discarded and each solution component is replaced by x new values generated by a random exploration process. If the worst solution sets are denoted by $x^{(m-x+1)}, x^{(m-x+2)}, \cdots, x^{(m)}$. The new solution components for the solution set $x^{(j)}$ are randomly generated as follows:

$$x^{(j)} = l_i + (u_i - l_i).rand_i^{(j)} \tag{22}$$

Where $i = 1, 2, \ldots, S$ and $j = (m-x+1),\ (m-x+2),\ \ldots, m$.

Random exploration ensures diversity and prevents premature convergence to local minima.

Pheromone Update

Initially, each solution component is assigned an initial pheromone value τ_o. The pheromone values are updated based on the quality of solutions constructed by the ants. The update is biased towards the best solutions constructed by the ants such that ACO concentrates the search in the regions of high quality solutions. Similar to MAX-MIN ant system [21], the pheromone values in SamACO algorithm are bounded to the interval $[T_{\min},\ T_{\max}]$; in this case $[0.1,\ 1]$.

The pheromones on the non-optimal paths are evaporated. The selected non-optimal solution components have their pheromones evaporated as

$$\tau_i^{(j)} \leftarrow (1-\rho)\tau_i^{(j)} + \rho\tau_{\min} \quad for \quad 0 < \rho < 1$$

$$i = 1, 2, \ldots, S \text{ and } j = 1, 2, \ldots, m \tag{23}$$

Where $\tau_{\min}$ is the predefined minimum pheromone value and ρ is the pheromone evaporation rate.

The pheromones on the near optimal paths are reinforced, thus influencing more ants to follow the paths and hopefully find better solutions. The solution components in the selected best Ψ solutions have their pheromone reinforced as

$$\tau_i^{(j)} \leftarrow (1-\beta)\tau_i^{(j)} + \beta\tau_{\max} \quad for \quad 0 < \beta < 1 \tag{24}$$

$$i = 1,2,\ldots,S \text{ and } j = 1,2,\ldots,\Psi$$

Where $\tau_{\max}$ is the predefined maximum pheromone value, β is the pheromone reinforcement rate, and Ψ is the elitist number.

In each iteration, the pheromone values of the solution components of the iteration-best are updated as the solution components of the best-so-far solution after the fitness evaluation of the m solutions constructed by the ants.

IMPLEMENTATION

Using MATLAB software, the proposed SamACO algorithm was implemented to compute the optimal switching angles that eliminate 5th, 7th, 11th, and 13th harmonics in an 11-level inverter. In this work, the population size is 40, and the number of iterations is 100. The solutions were computed by incrementing the modulation index, m_i in steps of 0.001 from 0 to 1. A personal computer (2.66 GHz Intel Core i7 processor with 4GB Random Access Memory) running MATLAB R2014b on OS X Yosemite version 10.10 was used to carry out the computations.

The solution set at each step is evaluated with the fitness function. The objective here is to determine the switching angles such that the selected low order harmonics are either eliminated or minimized to an acceptable level while the fundamental voltage is obtained at a desired value. For each solution set, the fitness function is calculated as follows [12]:

$$f = \min_{\alpha_i} \left[\left(100 \frac{V_1^* - V_1}{V_1^*} \right)^4 + \sum_{s=2}^{S} \frac{1}{h_s} \left(50 \frac{V_{hs}}{V_1} \right)^2 \right]$$

$$i = 1,2,\ldots,S \tag{25}$$

Subject to

$$0 \le \alpha_1 < \alpha_2 < \ldots < \alpha_s \le \pi/2$$

Where V_1^* is the desired fundamental output voltage, S is the number of switching angles, h_s is the order of the s^{th} viable harmonic at the output of a three phase multilevel converter. For example, $h_2 = 5$, $h_4 = 11$. It should be noted that different weight are assigned to different harmonics in eqn. (25). Each harmonic ratio is weighted by inverse of its harmonic order, i.e. $1/h_s$. By this weighting method, higher importance is assigned to the low order harmonics, which are more harmful and difficulty to remove with filter.

In order to validate the observed analytical results, an 11-level single-phase Cascaded H-Bridge inverter was modelled in MATLAB-SIMULINK using SimPower System block set. In each of the five H-Bridges in the 11-level single-phase Cascaded H-Bridge inverter, 12V dc source is the SDCS, and the switching device used is Insulated Gate Bipolar Transistor (IGBT). Simulations were performed at the fundamental frequency of 50 Hz using the solution set found at the modulation index, m_i of 0.795: $\alpha_1 = 6.99^o$, $\alpha_2 = 19.05^o$, $\alpha_3 = 28.01^o$, $\alpha_4 = 45.99^o$, and $\alpha_5 = 62.61^o$. Fast Fourier Transform (FFT) analysis of the simulated phase voltage waveforms was done using the FFT block to show the harmonic spectrum of the synthesized AC voltage.

RESULTS

The plots of fitness value for each set of switching angles versus modulation indices over the range of 0.1 to 1.0 is shown in Figure 3. For values of the fitness function less or equal to 0.01, SHE equations are solvable, otherwise they are unsolvable.

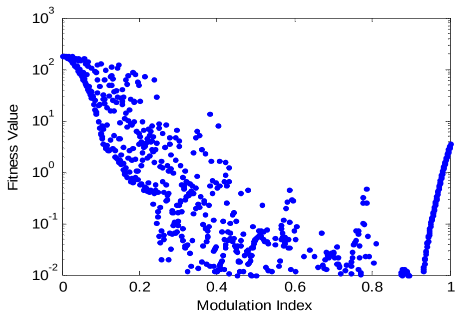


Figure 3: Fitness Function at Various Modulation Indices

As shown in Fig. 4, there are multiple solution sets at some modulation indices. In such cases, the solution set with the least THD value is chosen.

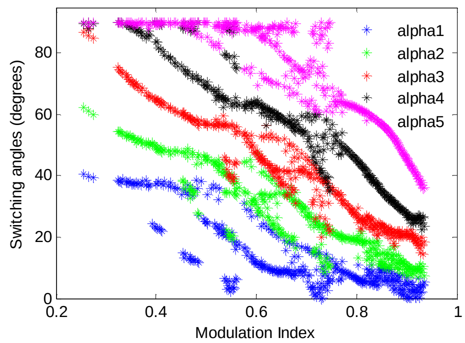


Figure 4: Switching Angles at Various Modulation Indices

At low modulation indices [0, 0.256], high modulation indices [0.934, 1] as well as some modulation indices, no solution sets are available.For those modulation indices, it is either there is no solution set or SamACO could not find one.

The former reason is more plausible than the latter.

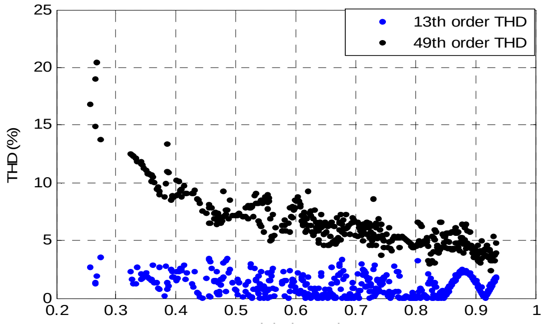


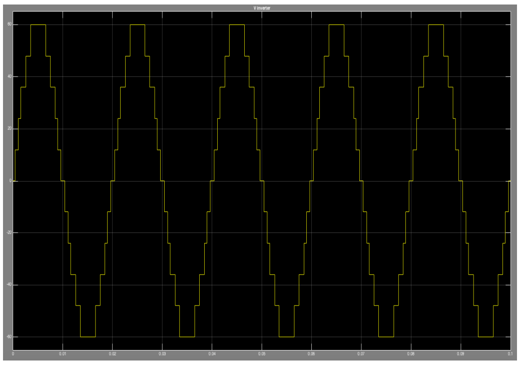
Figure 5: THD Values at Various Modulation Indices

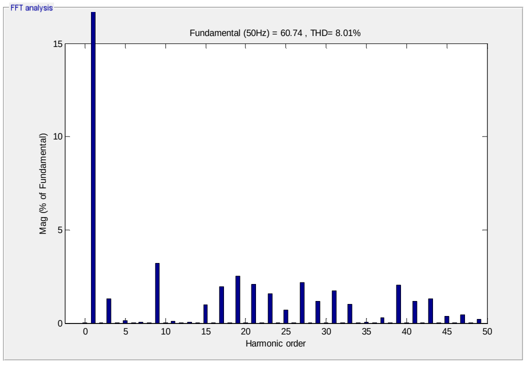
From the simulated output voltage waveform, the peak value of the fundamental output voltage is $60.74V_{(peak)}$. This value is in close agreement with the analytically computed value given by eqn. (8) as;

$$V_1 = m_i\left(\frac{4sV_{dc}}{\pi}\right) = 0.795\left(\frac{4\times5\times12}{\pi}\right) = 60.73V_{(peak)}$$

The FFT analysis of the synthesized voltage waveform shown in Fig.6 reveals the complete elimination of the $5^{th}, 7^{th}, 11^{th}$ *and* 13^{th} harmonics as their values tend towards zero.

The THD in line-to-line voltage as computed analytically and from simulation are 4.54% and 4.58% respectively. The analytical and simulation values of THD are in close agreement thereby validating the analytical results. The THD value of 8.01% shown in Fig. 6 is the THD value of the synthesized voltage waveform of a single phase inverter which includes triplen harmonic components.





(a) (b)

Figure 6: (a) Synthesized Waveform (b) Harmonic Spectrum of an 11-Level Inverter with $m_i = 0.795$

CONCLUSIONS

Variable Sampling Ant Colony Optimization (SamACO) algorithm with random initial values has been successfully implemented for solving the transcendental nonlinear equations characterizing the harmonics in an 11-level inverter. The proposed method is derivative-free, accurate and globally convergent. The absence of the selected harmonics in the synthesized output phase voltage validates the analytically computed results.

REFERENCES

1. K. A. Corzine, "Multi-Level Converters," The Handbook on Power Electronics, Edited by T.L. Skvarenina, CRC Press, 2002, pp. 6-1 - 6-23
2. R. H. Baker and L. H. Bannister, “Electric power converter,” U.S. Patent 3867643, Feb. 1975.
3. A. Nabae, I. Takahashi and H. Akagi, “A new neutral-point clamped PWM inverter,”*IEEE Trans. Ind. Applicat*., vol. IA-17, Sept./Oct. 1981, pp. 518–523.
4. T. A. Meynard and H. Foch, “Multi-level conversion: High voltage choppers and voltage- source inverters,” in *Proc. IEEE-PESC*, 1992, pp. 397–403.
5. P. Hammond, “A new approach to enhance power quality for medium voltage ac drives,” *IEEE Trans. Ind. Applicat.*, vol. 33, pp. 202–208, Jan./Feb. 1997
6. J. Rodríguez, J. Lai, F.Peng, “Multilevel inverters: a survey of topologies, controls and applications,” *IEEE Transactions on Industry Applications*, vol. 49, no. 4, Aug. 2002, pp. 724-738.
7. S. Khomfoi, L. M Tolbert, Chapter31.Multilevel Power Converters. The University of Tennessee.pp.31-1 to 31-50.
8. J. Kumar, B. Das, and P. Agarwal, “Selective Harmonic Elimination Technique for Multilevel Inverter,’’ 15th National Power System Conference (NPSC), IIT Bombay, 2008, pp. 608-613.
9. F. Swift and A. Kamberis, “A New Walsh Domain Technique of Harmonic Elimination and Voltage Control In Pulse-Width Modulated Inverters,” IEEE Transactions on Power Electronics, volume 8, no. 2, 1993, pp. 170–185.
10. J. N. Chiasson, L. M. Tolbert, K. J. McKenzie, and Z. Du, “Control of a Multilevel Converter Using Resultant Theory,’’ IEEE Transaction on Control Systems Technology, volume 11, no. 3, May 2003, pp. 345- 353.
11. B. Ozpineci, L. M. Tolbert, and J. N. Chiasson, “Harmonic Optimization of Multilevel Converters Using Genetic Algorithm,’’*35 Annual IEEE Power Electronics Specialists Conference*, Germany, 2004.
12. R. Salehi, N. Farokhia, M.Abedi, and S.H. Fathi, “Elimination of Low Order Harmonics in Multilevel Inverters Using Genetic Algorithm,’’ Journal of Power Electronics, volume 11, no. 2, Mar. 2011, pp. 132-139.
13. D.O.Aborisade, I.A.Adeyemo, and C.A, Oyeleye, “Harmonic Optimization in Multilevel Inverter Using Real Coded Genetic Algorithm,”International Journal of Electrical Engineering Research & Applications (IJEERA) volume 1, Issue 4, 2013.
14. K. Sundareswaran,K. Jayant, and T. N. Shanavas, “Inverter Harmonic Elimination through a Colony of Continuously Exploring Ants,” IEEE Transactions on Industrial Electronics, volume 54, no. 5, 2007, pp. 2558-2565.
15. A. Kavousi, et. al., “Application of the Bee Algorithm for Selective Harmonic Elimination Strategy in Multilevel Inverters,” IEEE Transaction on Power Electronics, vol. 27, no. 4, pp.1689-1696, April 2012.
16. N. Vinoth, and H. Umesh prabhu, ‘‘Simulation of Particle Swarm Optimization Based Selective Harmonic

Elimination," International Journal of Engineering and Innovative Technology (IJEIT) Volume 2, Issue 7, 2013, pp. 215-218.

17. S. Sirisukprasert, J. S. Lai, and T. H, Liu, "Optimum Harmonic with a Wide Range of Modulation Indices for Multilevel Converters," IEEE Transaction on Industrial Electronics, Vol. 49, no; 4, August 2002, pp. 875-881.
18. M. Dorigo, V. Maniezzo, and, A. Colorni, "Ant System: Optimization by a colony of Cooperating Agents.'' IEEE Transactions on Systems, Man, and Cybernetics, Part B, volume 26, no. 1, 1996, pp. 29-41.
19. M. Dorigo and L. Gambardella, "Ant Colony System: A cooperative Learning Approach to the Travelling Salesman Problem," IEEE Transactions on Evolutionary Computation, volume 1, no. 1, 1997, pp. 53-66.
20. X. Hu, J. Zhang, H. S. Chung, Y. Li, and O. Liu, "SamACO: Variable Sampling Ant Colony Optimization for Continuous Optimization,'' IEEE Transactions on Systems, Man, and Cybernetics, 40 (6), 2010, pp 1-36.
21. T. Stützle and H. H. Hoos, "MAX-MIN Ant System," Future Generation Computer Systems, volume 16, no. 8, 2000, pp. 889-914